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# A neutral anisotropic quark star model with conformal symmetry

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## Abstract

New solution for the Einstein field equations satisfying a neutral anisotropic quark star object is generated. The quark linear equation of state together with the conformal Killing vector (CKV) are used to investigate the behavior and properties of quark stars. The CKV plays an important role in providing the relationship between the two gravitational potentials. The process of combining the CKV and an equation of state has currently led to new realistic solutions. In obtaining the matter variables, one of the gravitational potentials is specified on physical grounds to generate compact star model with physical significance. The generated quark star model undergoes several physical tests for the validity and acceptability. Several realistic physical conditions are found to be satisfied. The model stability in terms of the adiabatic index and equilibrium of the physical forces is satisfied. The behavior of the mass-radius relationship and the surface red shift are well obeyed. Their parameters values are found to be compatible with observations. The gravitational potentials are continuous throughout the interior of the star, and the model's energy conditions are satisfied as well. Quark star models that admit conformal symmetry in the absence of charge are missing in the existing literature.

**Keywords:** *Conformal Killing vector; Einstein field equations; Equation of State; Pressure anisotropy; Quark Star*

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## Introduction

Equations of state have been fruitful in describing the material compositions within relativistic stars. After the massive stars undergone evolution when the nuclear forces fail to balance with the natural gravitational forces, several compact objects are formed including white dwarfs, neutron stars, black holes and gravastars (Joshi, 2007). The nature of material compositions within these objects are best explained by equations of state including the quadratic, polytropic, Van der Waals and linear equations of state. A number of studies on the useful of quadratic equation of state in generating relativistic models with physical significance have been carried out by various researchers. Sunzu and Thomas (2018) generated realistic exact solutions for uncharged star model with vanishing pressure anisotropy using a quartic equation of state. Sunzu and Mathias (2022) used a quadratic equation of state to study the behavior and properties of a neutral star with a choice of the gravitational potential that regains several models found by other researchers. Pant and Fuloria (2021) conducted a comparative analysis of a charged anisotropic compact stellar model by considering quadratic and linear equations of state. Models with polytropic equations of state have been studied by several researchers. Afzal and Feroze (2024) generated new classes of exact solutions for charged anisotropic compact stars by varying different polytropic indices where a comprehensive analysis of a neutron star PSR J0740+6620 was provided. Thirukkanesh *et al*. (2020) analyzed the behavior of anisotropic compact star exhibiting a paraboloidal geometry with a polytropic equation of state where the radial pressure found to dominate the tangential pressure throughout the interior of the star. Van der Waals equations of state have been used by various researchers to generate realistic compact star models with physical significance (Ditta *et al.,* 2023; Errehymy *et al.,* 2022). Relativistic studies on linear equations of state have been conducted by various researchers. Patel *et al*. (2023) generated exact solutions of Einstein's field equations compatible with several stars like 4U 1820-30, PSR J1903+327, EXO 1785-248, LMC X-4 by considering a linear equation of state with a surface density term. Jape *et al*. (2023a) investigated the behavior and properties of charged anisotropic compact stars with a conformal Killing vector and a linear equation of state where a number of solutions found by other researchers in the literature are contained.

The role of pressure anisotropy in investigation the behavior of compact star models has been an important area of consideration. As first pointed out by Herrera and Santos (1997), it has been found that various factors contributed to the sources of pressure anisotropy in relativistic fluids. The presence of strong electromagnetic field in these highly dense fluids may influence pressure anisotropy (Martinez *et al.,* 2003; Usov, 2004). Transitions in phases within compact objects may influence pressure anisotropy as well (Herrera and Nunez, 1989; Sokolov, 1980). It has been also found by Kippenhahn and Weigert (1990) that the presence of type-3A superfluid in relativistic fluid can be the source of anisotropy. The effects of pressure anisotropy within relativistic fluids have been analyzed in various studies. Ruderman (1972) observed that pressure anisotropy can influence higher density values ($> 10^{15}\ gcm^{-3}$) within relativistic fluids. Herrera and Santos (1997) found the influence of pressure anisotropy on the stability of compact stars. Bowers and Liang (1974), and Dev and Gleiser (2002) observed variations of mass, surface redshift and mass radius ratio for fluid spheres with anisotropic nature. Other effects of pressure anisotropy on relativistic objects are found in various researches (Herrera, 2020; Chan *et al.,* 1993; Cosenza *et al.,* 1981). This study aims to develop a neutral realistic quark star model with anisotropic nature throughout the interior of the star.

Several approaches have been used by various researchers in analyzing the behavior and properties of relativistic compact objects. The use of embedding of dimensions proved to be efficient in investigating these objects. Mathias *et al.* (2021) used embedding of dimensions to generate a charged anisotropic model with a generalized measure of anisotropy that contains several models found by other researchers. Maurya *et al.* (2021) obtained a solution of Einstein-Maxwell field equations via embedding of dimensions for a charged anisotropic star describing several compact objects like PSR J1903+327; Cen X-3; EXO1785-248 and LMCX-4. This was done for a massive scalar field under the Brans-Dicke gravity. The modified theories of gravitation have been used to study compact objects as well. The $f(R,T)$ theory plays an important role in understanding the behavior of compact objects. Harko *et al.* (2011) introduced this theory to obtain the gravitational field equations in metric form where by an arbitrary function of the Ricci scalar and the stress-energy tensor was used to define the gravitational Lagrangian. Tangphati *et al.* (2023) used the higher dimensional modified $(R,T)$ gravity theory to study charged strange quark stars by assuming a linear relationship between the fluid energy density and charge density for numerical computation purposes. Other modified gravity theories like Einstein-Gauss-Bonnet (Jasim *et al.,* 2021; Maurya *et al.,* 2020; Hansraj *et al.,* 2019) and $f(R)$ gravity (Nashed and Capozziello, 2021; Abbas *et al.,* 2015; Ilyas and Ahmad, 2024) have been useful in studying behavior and properties of compact stars.

Several studies have considered the approaches of using the quark linear equation of state and the conformal symmetry separately. Models of compact fluid spheres admitting conformal symmetry have been investigated by several authors. Jape *et al.* (2021) obtained a generalized compact star model by solving the conformal condition through specifying specific forms for the electric charge and the measure of anisotropy where several famous known models of Finch and Skea (1989) and Vaidya and Tikekar (1982) are regained. Jape *et al.* (2023b) used conformal Killing vector to generate a charged anisotropic solution for compact star without employing an equation of state. Maurya *et al.* (2019) applied the conformal Killing vector to investigate the behavior of a neutral star that satisfies various physical relativistic conditions. Christopher *et al.* (2024) generated solutions to the Einstein-Maxwell field equations for a charged conformal star with a quadratic equation of state. Studies on quark matter with linear equations of state are found in various investigations (Cheng *et al.*, 2010; Klahn *et al.*, 2007; Rincon *et al.*, 2023; Mak and Harko, 2002). This paper is motivated to investigate the behavior and properties of a strange quark star with a linear equation of state and conformal symmetry. A linear equation of state for quark matter and conformal Killing vector are simultaneously used to obtain the gravitational potentials and other matter variables for analysis. The study on uncharged strange quark star that admit conformal symmetry is missing in the current observations. This paper is organized in the following fashion: in the next section we provide the materials and methods used. In Section 3 we provide the results of our investigation, and the discussion of the results is given in Section 4. Section 5 provides the study conclusion while the last section outlines the study recommendations.

## Materials and Methods

This section outlines important materials used in this study as well as the methods of approach. The important equations governing the investigation are clearly presented and explained. This provides foundations in generating the required model for analysis.

### *Einstein Field Equations*

The interior (Schwarzschild) and exterior (Reissner-Nordstrom) lines elements are used in formulating the Einstein field equations for a highly gravitating uncharged star characterized by high density and pressures. These are respectively given by the equations.

$$ds^2 = r^2(d\varphi^2 + sin^2\varphi d\theta^2) + e^{2\lambda}dr^2 - e^{2\nu}dt^2 \tag{1}$$

and

$$ds^2 = r^2(d\varphi^2 + sin^2\varphi d\theta^2) + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - \left(1 - \frac{2M}{r}\right) dt^2, \tag{2}$$

where $\nu$ and $\lambda$ are gravitational potentials and $M$ stands for the mass of the compact obect. To obtain the field equations for the study, we present the Einstein and the energy momentum tensors as

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta}, \tag{3}$$

and

$$T_{\alpha\beta} = (\rho + p_t)\, u_\alpha u_\beta + p_t\, g_{\alpha\beta} + (p_t - p_r)v_\alpha v_\beta, \tag{4}$$

respectively, where $\rho$, $p_t$ and $p_r$ are respectively the fluid energy density, the transverse pressure, and the radial pressure. The quantity $g_{\alpha\beta}$ stands for the metric tensor. Combining equations (1), (2), (3) and (4) with some mathematics, the Einstein field equations for a neutral star model are obtained and given by

$$\rho(r) = e^{-2\lambda}\left(\frac{2\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}, \tag{5a}$$

$$p_r(r) = e^{-2\lambda}\left(\frac{2\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}, \tag{5b}$$

$$p_t(r) = e^{-2\lambda}\left(\nu'' + \nu'^2 - \nu'\lambda' + \frac{\nu'}{r} - \frac{\lambda'}{r}\right). \tag{5c}$$

The primes in system (5) indicate the derivatives of the respective quantities with respect to the radial distance $r$. The measure of anisotropy $\Delta$ is obtained by subtracting equation (5b) from equation (5c). When $\Delta \neq 0$, which happens when the tangential pressure is different from the radial pressure, we speak about anisotropic compact star models. For isotropic models, the radial pressure is always the same as the tangential pressure.

The mass equation for the uncharged fluid star as given by Mak and Harko (2003) and Sunzu *et al.* (2014) is defined to be

$$M(r) = \frac{1}{2}\int_0^r \rho(\omega)\omega^2 d\omega. \tag{6}$$

***Conformal Symmetry***

We present the conformal Killing vector and analyze its importance in simplification of the field equations. It is observed that the Einstein field equations (5) are highly non-linear and therefore difficult to solve. To simplify these equations, the conformal Killing vector is introduced which helps to provide an important relationship between the gravitational metric functions. This vector is given by

$$L\boldsymbol{X}g_{\boldsymbol{\alpha\beta}} = 2\gamma g_{\alpha\beta}, \tag{7}$$

where $\boldsymbol{X}$ is the conformal Killing vector and $\gamma$ is the conformal factor that preserve the metric of space-time. Here $L$ is the derivative operator for lie algebra on the conformal vector. We consider the non-static condition for the conformal vector as well as its associated conformal factor as outlined in Jape *et al.,* (2021). These are respectively given by

$$X = \alpha(t,r)\frac{\partial}{\partial t} + \beta(t,r)\frac{\partial}{\partial r}, \tag{8a}$$

$$\gamma = \gamma(t,r). \tag{8b}$$

Equation (7) can be solved by utilizing the Killing vector (8a) and the conformal factor (8b) together with the specific associated integrability condition. This condition is given by the Weyl tensor

$$L\boldsymbol{X}C^a_{bcd} = 0. \tag{9}$$

In equation (9), $C^a_{bcd}$ stand for the non-zero values of the Weyl tensor. Now, using the system of equations (8) with condition (9), the vector equation (7) reduces to the highly non-linear differential equation

$$e^{-2\lambda}\left(\nu'' + \nu'^2 - \nu'\lambda' + \frac{\lambda'}{r} - \frac{\nu'}{r} + \frac{1}{r^2}\right) = \frac{(1+k)}{r^2}. \tag{10}$$

Solving equation (10) yields

$$e^{\nu} = \begin{cases} Arexp\left(\sqrt{1+k}\int\frac{e^{\lambda}}{r}dr\right) + Brexp\left(-\sqrt{1+k}\int\frac{e^{\lambda}}{r}dr\right), & 1+k>0 \\ Ar\int\frac{e^{\lambda}}{r}dr + Br, & 1+k=0 \\ Arexp\left(\sqrt{-(1+k)}\int\frac{e^{\lambda}}{r}dr\right) + Brexp\left(-\sqrt{-(1+k)}\int\frac{e^{\lambda}}{r}dr\right), & 1+k<0. \end{cases} \tag{11}$$

In equation (11), $A$ and $B$ are constant parameters. This is an important equation relating the two gravitational potentials $e^{\lambda}$ and $e^{\nu}$. It is observed that the specification of the gravitational potential $e^{\lambda}$ on physical grounds can help us to integrate equation (11) for different cases and obtain the second gravitational potential $e^{\nu}$. It is important to understand that when $k = 0$ in equation (11), we then define a conformally flat space-time.

***The quark equation of state***

We define the quark linear equation of state for quark matter as

$$p_r = \frac{1}{3}(\rho - 4\gamma), \tag{12}$$

where $\gamma$ is constant. The quark equation of state (12) was also used by various researchers to generate realistic models with physical significances (Abdalla *et al.,* 2021; Singh *et al.,* 2021; Mathias *et al.,* 2024). This was done without imposing the conformal condition on the space-time manifold. Jape *et al.* (2023a) used a general linear equation of state and conformal Killing vector to investigate the behavior of a charged compact star. For this investigation, the quark equation of state (12) is treated in the presence of the conformal Killing vector to study the behavior of uncharged quark star. This helps to generate new realistic exact solutions for uncharged quark stars.

***Transformation of the field equations and the conformal condition***

The field equations (5), the mass function (6) and the conformal condition (11) are transformed for simplification purposes. This is done by using the Durgapal and Bannerji (1983) transformations given by

$$x = Cr^2,\ z(x) = e^{-2\lambda},\ \ y^2(x) = e^{2\nu}. \tag{13}$$

Mathematical simplification of system (5) through using transformations (13) leads to the system of Einstein field equations

$$\frac{\rho(x)}{C} = \frac{1-z}{x} - 2\dot{z}, \tag{14a}$$

$$\frac{p_r(x)}{C} = \frac{z-1}{x} + 4z\frac{\dot{y}}{y}, \tag{14b}$$

$$\frac{p_t(x)}{C} = \dot{z} + 4xz\frac{\ddot{y}}{y} + (4z + 2x\dot{z})\frac{\dot{y}}{y}. \tag{14c}$$

$$\frac{\Delta(x)}{C} = \frac{1-z}{x} + 4xz\frac{\ddot{y}}{y} + 2x\dot{z}\frac{\dot{y}}{y} + 1. \tag{14d}$$

The dots in system (14) are for the differentiation of the given functions with respect to the radial coordinate $x$ while $C$ is a constant parameter. The anisotropic equation (14d) is obtained by subtracting equation (14b) from (14c). Likewise, using the same transformations (13), the conformal condition (11) becomes

$$y = \begin{cases} A\sqrt{x}\, exp\left(\frac{1}{2}\sqrt{(2n-1)}\int\frac{dx}{x\sqrt{z}}\right) + B\sqrt{x}\, exp\left(-\frac{1}{2}\sqrt{(2n-1)}\int\frac{dx}{x\sqrt{z}}\right), & n > \frac{1}{2} \\ \frac{A}{2}\sqrt{x}\int\frac{dx}{x\sqrt{z}} + B\sqrt{x}, & n = \frac{1}{2} \\ A\sqrt{x}\, exp\left(\frac{1}{2}\sqrt{-(2n-1)}\int\frac{dx}{x\sqrt{z}}\right) + B\sqrt{x}\, exp\left(-\frac{1}{2}\sqrt{-(2n-1)}\int\frac{dx}{x\sqrt{z}}\right), & n < \frac{1}{2}. \end{cases} \tag{15}$$

In this work, the conformal condition (15) is used with the quark linear equation of state (12) to generate a neutral compact star model for quark matter. A thorough physical analysis is then carried out to test the validity and acceptability of the generated class of exact solutions.

**Results**

In this section, we present the resulting quark star model generated by merging the conformal Killing vector and a quark linear equation of state. The quark linear equation of state (12) and the conformal condition (15) are used together with the Einstein field equations (14) to obtain the gravitational potentials and other matter variables. From condition (15), we simply consider the case $n = \frac{1}{2}$ with the metric function $y$ being

$$y = \frac{A}{2}\sqrt{x}\int\frac{dx}{x\sqrt{z}} + B\sqrt{x}. \tag{16}$$

To integrate (16), we opt to specify the gravitational potential $z$. This specification is done on physical grounds for the purpose of obtaining the second gravitational potential $y$. This is done by choosing

$$z = \left(\frac{1+ax}{1-bx}\right)^2. \tag{17}$$

The metric function (17) is physical and realistic as it is noted that $z = 1$ at the stellar interior ($x = 0$) as proposed by various researchers (Errehymy *et al.,* 2022; Jape *et al.,* 2023a; Herrera and Santos, 1997). This condition is important for realistic quark star models. Similar metric functions to (17) were used by various researchers to study the behavior of compact stars using approaches other than the conformal symmetry (Olengeile *et al.,* 2023, 2024; Mathias and Sunzu, 2022). However, in this study, the metric function (17) is used with conformal Killing vector to study the behavior and properties of a neutral quark star.
To obtain the matter variables, we first use equation (17) into (16) to get the second gravitational potential as

$$y = \sqrt{x}\left(\frac{A}{2}(a\log(x) - (a+b)\log(1+ax)) + B\right). \tag{18}$$

Now, using equations (12), (17) and (18) with system (14), the matter variables become

$$\rho = C\left(a^2x(5-bx)+2a(3+bx)+b(6-3bx+b^2x^2)\right)(-1+bx)^{-3}, \tag{19a}$$

$$p_r = \frac{1}{3}\left(C\left(a^2x(5-bx)+2a(3+bx)+b(6-3bx+b^2x^2)\right)(-1+bx)^{-3}-4\gamma\right), \tag{19b}$$

$$\begin{aligned} p_t = &-C((2(1+ax)(-3aA-2B-2aB\sqrt{x}-2bB\sqrt{x}+6abAx-4aBx-3aAb^2x^2 \\ &+2abBx^2+aA\left(-1-b\sqrt{x}+a\left(-\sqrt{x}-2x+bx^2\right)\right)\log(x)+A(a+b(1+b\sqrt{x}) \\ &-ab^2x^2+2ab\left(\sqrt{x}+x\right)+a^2(\sqrt{x}+2x-bx^2))\log(1+ax)))(\sqrt{x}(-1+bx)^3 \\ &(-2B-aA\log(x)+A(a+b)\log(1+ax)))), \end{aligned} \tag{19c}$$

$$\Delta = p_t - p_r. \tag{19d}$$

The system of equations (19) provides the model equations for the quark star. This is an important set of equations that gives the analysis for the behavior and properties of quark matter. These model equations determine whether the generated class of exact solution represent astrophysical model with physical significance. Also, using the energy density equation (19a), the mass function (6) in radial coordinate $x$ becomes

$$M(x) = \frac{-(a+b)C^{\frac{1}{2}}x^{\frac{3}{2}}(2+ax-bx)}{2(-1+bx)^2}. \tag{20}$$

The mass function (20) is important because it helps to understand the behavior of other properties including the mass-radius ratio and the surface redshift.

## Discussion

This section discusses different properties and behavior of the resulting quark star model. The physical relativistic conditions are analyzed to identify whether the generated class of exact solution presents a relativistic quark star model with physical significances. This is done by comparing the results obtained to other models generated by various researchers in the literature. This is important for model validity and acceptability. In conducting the analysis for the study and generating the plots, we have specified the values of the model parameters as $a = 0.005$, $b = -0.350$, $C = 0.050$, $A = 1.005$, $B = 0.050$, and $\gamma = 0.0186$. This was done so that a well behaved model is obtained. In plotting the graphs, the *Python* programing was opted while *Mathematica* was used for computational purposes.

### *Model's stability conditions*

The stability of the generated quark star model is analyzed by studying the behavior of the heating effects within the star. This is given by the ratio of the specific heat capacity to constant pressure to that of constant volume. This is analyzed by the adiabatic conditions

$$\Gamma = \frac{(\rho+p_r)}{p_r}\frac{p_r{}'}{\rho'}, \tag{21a}$$

$$\Gamma_{crit} = \frac{4}{3}+\frac{19}{21}\mu, \tag{21b}$$

where $\mu$ is the mass-radius ratio. It is required that the conditions (21) satisfy the requirement $\Gamma \geq \Gamma_{crit.} \geq \frac{4}{3}$ (Maurya and Nag, 2021; Tello-Ortiz *et al.*, 2020). The generated quark star model satisfies this condition as illustrated in **Figure 1**. Similar features are also found in several investigations (Jape *et al.*, 2021; Mathias *et al.*, 2024). The stability in terms of cracking condition is also analyzed and it was found that the speed of sound $v_r{}^2$ is less than that of light (i.e. $v_r{}^2 < 1$) as indicated in the same figure. The satisfaction of these conditions show that the generated class of solutions represent a realistic quark start model.

### *Equilibrium of the physical forces*

The total interior forces within the star need to balance for equilibrium requirement. This condition is outlined in Tolman-Oppenheimer-Volkoff (TOV) equation (Oppenheimer and Volkoff, 1939) given by

$$\frac{-\nu'(\rho+p_r)}{2} - \frac{dp_r}{dr} + \frac{2\Delta}{r} = 0. \quad (22)$$

The consecutive terms in equation (22) stand for the gravitational ($F_g$), hydrostatic ($F_h$) and anisotropic ($F_a$) forces, respectively. The sum of these forces are required to be zero for equilibrium requirement, that is $F_g + F_h + F_a = 0$ (Rej *et al.,* 2021; Naz *et al.,* 2021). The profiles for the behavior of the forces are indicated in **Figure 2**. It is clear that the equilibrium condition is satisfied as the forces sum up to zero. Similar trends are observed in various studies (Jape *et al.,* 2023a; Olengeile *et al.,* 2023).

***Continuity of the metric functions***

The metric functions $y$ and $z$ are required to be continuous and free from central singularities. This is an important condition for the regularity of these quantities. The gravitational potentials are required to be $y = e^{2\nu} \geq 0$ and $z = e^{-2\lambda} = 1$ at the stellar center (Errehymy *et al.,* 2022; Patel *et al.,* 2023; Nashed and Capozziello, 2021). It is observed from **Figure 3** that the behavior of the gravitational potentials satisfies this requirement. Similar observations are found in various researches (Jasim *et al.,* 2021; Thirukkanesh *et al.,* 2020).

***Energy conditions***

The conditions for the energy properties within a quark star need to be satisfied. The energy conditions include the weak energy condition (WEC), the dominant energy condition (DEC) and the strong energy condition (SEC). These conditions are given by the following equations:

$$\text{WEC:} \rho - p_r \geq 0;\ \rho - p_t \geq 0, \quad (23a)$$

$$\text{DEC: } \rho - 3p_r \geq 0;\ \rho - 3p_t \geq 0, \quad (23b)$$

$$\text{SEC: } \rho - p_r - 2p_t \geq 0. \quad (23c)$$

All these conditions require the energy to be positive throughout the stellar interior (Sunzu and Mathias, 2022; Thirukkanesh *et al.,* 2020; Maurya *et al.,* 2021). This condition is satisfied as indicated in **Figure 4**. Similar profiles were also generated by several authors (Patel *et al.,* 2023; Mathias *et al.,* 2021; Maurya *et al.,* 2019).

***Mass, Surface redshift and Compactness***

The mass of the quark star should be an increasing function from the center to the surface. It is also required that for anisotropic fluids, the mass-radius ratio defining the compactness factor to be $\mu = \frac{M}{r} \leq 0.587$ and the surface redshift $z_s = -1 + (1-2\mu)^{-\frac{1}{2}}$ not exceeding 5.211 (Buchdahl, 1959; Ivanov, 2002). The profiles for these variables are shown in **Figure 5**. The maximum value for compactness factor is found to be $\mu = 0.259$ and that for surface redshift is $z_s = 0.167$. These values are compatible with several observations in the literature. It is clear that the generated quark star model is physical and realistic. Similar profiles were also generated by various researchers in the past (Jape *et al.,* 2023b; Christopher *et al.,* 2024).

**Conclusion**

In this study, a new solution for uncharged quark star admitting conformal symmetry was generated. A quark linear equation of state was used with a conformal Killing vector to study the behavior and properties of quark matter. The conformal condition was analyzed by specifying one of the gravitational potentials on physical grounds to get the second. This process together with the application of the quark equation of state and the Einstein field equations help to obtain the matter variables for analysis. It is interesting that we managed to generate a realistic quark star model with conformal Killing vector that satisfies many important physical conditions. Uncharged compact star models with quark matter composition that admit conformal symmetry are missing in the current literature. A detailed analysis of the physical relativistic conditions was conducted to test the validity and acceptability of the generated class of exact solution. It was found that the stability condition under the adiabatic index ratio and the causality condition for the speed of sound throughout the stellar interior is satisfied. The speed of sound was found to be one third of the speed of light. This condition must be satisfied for all quark matters. The gravitational potentials were found to be regular, continuous throughout the interior of the quark star and without central singularities. The interior natural physical forces balanced for equilibrium requirement as the total forces sum up to zero. It was also found that all the necessary energy conditions were satisfied as well. The behavior of these energies were shown to be positive throughout the star interior. We have also investigated the behavior of the mass of the star and the mass-radius ratio as well as the surface redshift. It was observed that the values of all these variables are found within the acceptable ranges for compact stars. Satisfaction of all these conditions indicate that our class of exact solution for a quark star is physically realistic.

**Recommendations**

In this work, we have limited ourselves to investigate the behavior and properties of a quark star that admits conformal symmetry. This helped us to generate exact solution of Einstein field equations for uncharged quark star. We have not engaged to study properties of stars with other matter composition described by different equations of state like polytropic, Chaplygin, or Van der Waals. It is therefore recommended that on the future one can study the properties of compact stars with different material compositions by using either polytropic, Chaplygin or Van der Waals equations of state in the presence of conformal symmetry. This can be done by specifying new forms for one of the gravitational potentials on physical basis. This process may lead to generation of realistic compact star models with physical significances.

**Figure 1**

*Adiabatic indices and sound speed against radial coordinate $r$.*

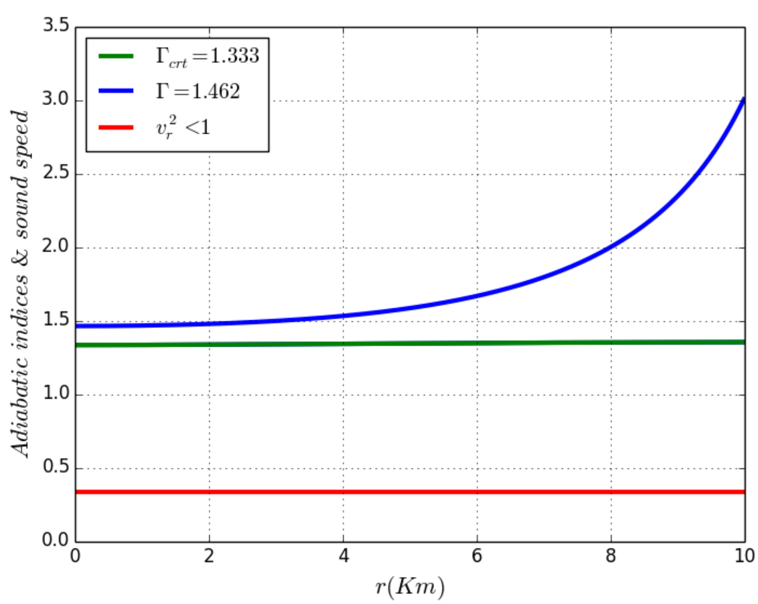


**Figure 2**

*Equilibrium forces against radial coordinate* $r$.

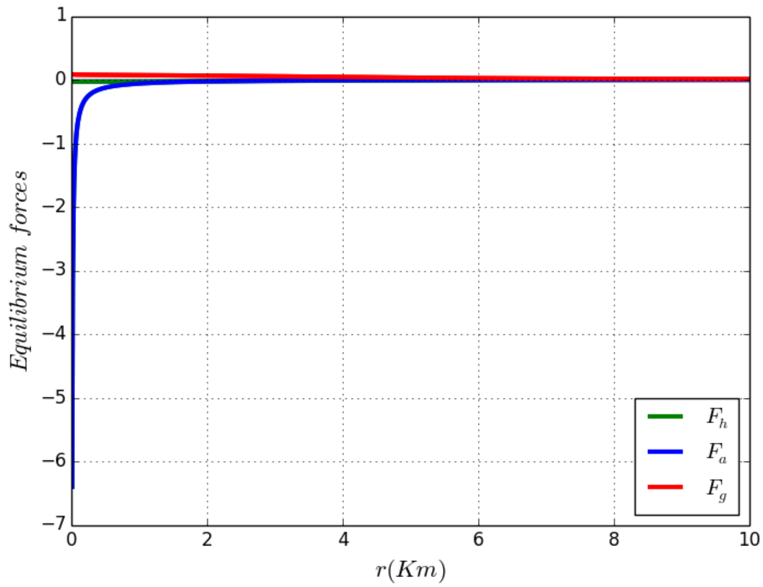



**Figure 3**

*Gravitational potentials against radial coordinate* $r$.

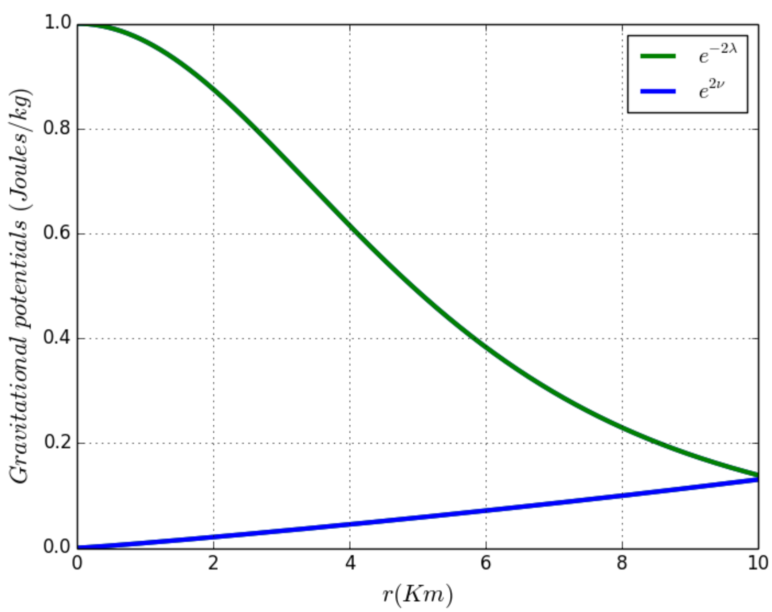


**Figure 4**

*Energy conditions against radial coordinate r.*

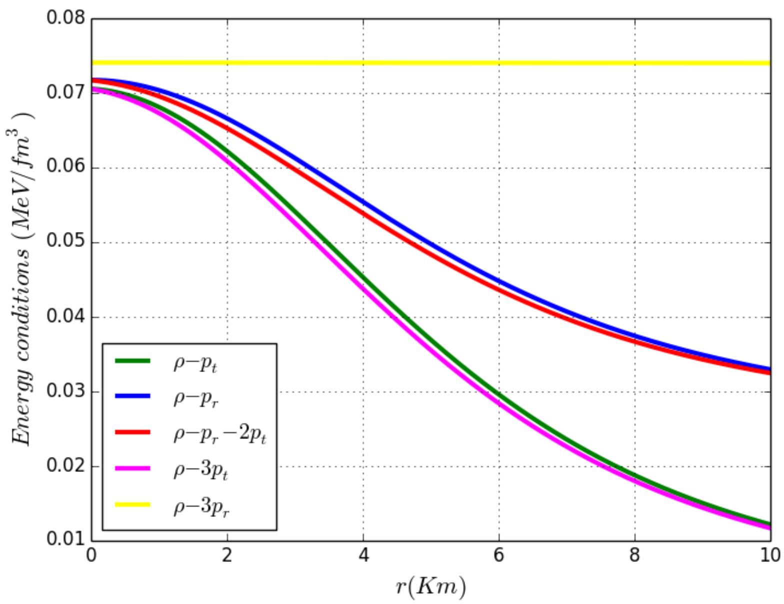
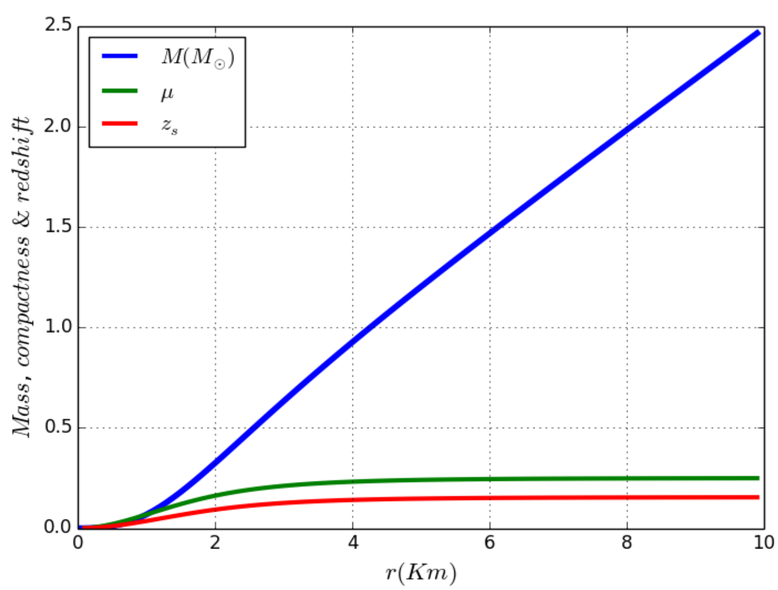



**Figure 5**

*Mass, compactness and surface redshift against radial coordinate r*



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**References**

Abbas, G., Zubair, M., & Mustafa, G. (2015). Anisotropic strange quintessence stars in f (R) gravity. *Astrophysics and Space Science*, *358*(2), 26.

Abdalla, A. T., Sunzu, J. M., & Mkenyeleye, J. M. (2021). Generalised charged anisotropic quark star models. *Pramana*, *95*(2), 86.

Afzal, W. A., & Feroze, T. (2024). A generalized polytropic equation of state stellar model with anisotropic charged fluid. *Astrophysics and Space Science,* 369, 11.

Bowers, R. L. & Liang, E. P. T. (1974). Anisotropic Spheres in General Relativity. *Astrophysical Journal*, 188, 657.

Buchdahl, H. A. (1959). General relativistic fluid spheres. *Physical Review*, *116*(4), 1027.

Chan, R., Herrera, L., & Santos, N. O. (1993). Dynamical instability for radiating anisotropic collapse. *Monthly Notices of the Royal Astronomical Society*, 265(3), 533–544.

Cheng, M., Ejiri, S., Hegde, P., Karsch, F., Kaczmarek, O., Laermann, E., ... & Soeldner, W. (2010). Equation of state for physical quark masses. *Physical Review D*, *81*(5), 054504.

Christopher, J., Jape, J. W., & Sunzu, J. M. (2024). Charged Anisotropic Conformal Star Model with a Quadratic Equation of State. *International Journal of Modern Physics D*. 33(01), 2450002.

Cosenza, M., Herrera, L., Esculpi, M., & Witten, L. (1981). Some models of anisotropic spheres in general relativity. *Journal of Mathematical Physics,* 22, 118–125.

Dev, K., & Gleiser, M. (2002). Anisotropic stars: exact solutions. *General relativity and gravitation*, *34*, 1793-1818.

Ditta, A., Tiecheng, X., Errehymy, A., Mustafa, G., & Maurya, S. K. (2023). Anisotropic charged stellar models with modified Van der Waals EoS in f (Q) gravity. *The European Physical Journal C*, *83*(3), 254.

Durgapal, M. C., & Bannerji, R. (1983). New analytical stellar model in general relativity. *Physical Review D*, *27*(2), 328.

Errehymy, A., Mustafa, G., Khedif, Y., Daoud, M., Alrebdi, H. I., & Abdel-Aty, A. H. (2022). Self-gravitating anisotropic model in general relativity under modified Van der Waals equation of state: a stable configuration. *The European Physical Journal C*, *82*(5), 1-11.

Finch, M. R., & Skea, J. E. (1989). A realistic stellar model based on an ansatz of Duorah and Ray. *Classical and Quantum Gravity*, *6*(4), 467.

Hansraj, S., Maharaj, S. D., & Chilambwe, B. (2019). Constant potentials in 6D Einstein-Gauss-Bonnet theory. *Physical Review D*, *100*(12), 124029.

Harko, T., Lobo, F. S., Nojiri, S. I., & Odintsov, S. D. (2011). f (R, T) gravity. *Physical Review D, 84*(2), 024020.

Herrera, L. (2020). Stability of the isotropic pressure condition. *Physical Review D*, 101, 104024.

Herrera, L. & Nunez, L. (1989). Modeling 'hydrodynamic phase transitions' in a radiating spherically symmetric distribution of matter. *Astrophysical Journal*, 339, 339-353.

Herrera, L., & Santos, N. O. (1997). Local anisotropy in self-gravitating systems. *Physics Reports,* 286(2), 53-130.

Ilyas, M., & Ahmad, D. (2024). Stability analysis of anisotropic stellar structures in f (R) gravity. *Chinese Journal of Physics*, 88, 901-912.

Ivanov, B. (2002). Static charged perfect fluid spheres in general relativity. *Physical Review D, 65*(10), 104001.

Jape, J. W., Maharaj, S. D., Sunzu, J. M., & Mkenyeleye, J. M. (2023b). Charged anisotropic fluid spheres with conformal symmetry. *Indian Journal of Physics*, *97*(6), 1655-1671.

Jape, J. W., Maharaj, S. D., Sunzu, J. M., & Mkenyeleye, J. M. (2021). Generalized compact star models with conformal symmetry. *The European Physical Journal C, 81*, 1-12.

Jape, J. W., Sunzu, J. M., Maharaj, S. D., & Mkenyeleye, J. M. (2023a). Charged conformal stars and equation of state. *Indian Journal of Physics,* 97, 1015–1026.

Jasim, M. K., Maurya, S. K., Singh, K. N., & Nag, R. (2021). Anisotropic Strange Star in 5 D Einstein-Gauss-Bonnet Gravity. *Entropy*, *23*(8), 1015.

Joshi, P. (2007). *Gravitational Collapse and Spacetime Singularities*. Cambridge University Press, New York.

Kippenhahn, R. & Weigert, A. (1990). *Stellar Structure and Evolution* (Springer, Berlin).

Klähn, T., Blaschke, D., Sandin, F., Fuchs, C., Faessler, A., Grigorian, H., ... & Trümper, J. (2007). Modern compact star observations and the quark matter equation of state. *Physics Letters B, 645* (5-6), 170-176.

Mak, M. K., & Harko, T. (2003). Anisotropic stars in general relativity. *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, 459* (2030), 393-408.

Mak, M. K., & Harko, T. (2002). An exact anisotropic quark star model. *Chinese Journal of Astronomy and Astrophysics*, *2*(3), 248.

Martínez, A., Rojas, H., & Cuesta, H. (2003). Magnetic collapse of a neutron gas: Can magnetars indeed be formed? *The European Physical Journal C,* 29, 111–123.

Mathias, A. K., Maharaj, S. D., Sunzu, J. M., & Mkenyeleye, J. M. (2021). Charged anisotropic models via embedding. *Pramana Journal of Physics,* 95, 178.

Mathias, A. V., & Sunzu, J. M. (2022). A Well-Behaved Anisotropic Strange Star Model. *Advances in Mathematical Physics*, *2022*(1), 7243750.

Mathias, A. V., Sunzu, J. M., & Mkenyeleye, J. M. (2024). Core-envelope anisotropic star model admitting Karmarkar condition. *New Astronomy*, *106*, 102115.

Maurya, S. K., Maharaj, S. D., & Deb, D. (2019). Generalized anisotropic models for conformal symmetry. *The European Physical Journal C*, *79*, 1-15.

Maurya, S. K., & Nag, R. (2021). MGD solution under Class I generator. *The European Physical Journal Plus*, *136*(6), 1-34.

Maurya, S. K., Singh, K. N., Govender, M., Errehymy, A., & Tello-Ortiz, F. (2021). Anisotropic stars via embedding approach in Brans–Dicke gravity. *The European Physical Journal C*, *81*(8), 729.

Maurya, S. K., Tello-Ortiz, F., & Jasim, M. K. (2020). An EGD model in the background of embedding class I space–time. *The European Physical Journal C*, *80*(10), 918.

Nashed, G. G. L., & Capozziello, S. (2021). Anisotropic compact stars in f (R) gravity. *The European Physical Journal C*, *81*(5), 1-20.

Naz, T., Usman, A., & Shamir, M. F. (2021). Embedded class-I solution of compact stars in f (R)gravity with Karmarkar condition. *Annals of Physics*, *429*, 168491.

Olengeile, L., Sunzu, J. M., & Mkenyeleye, J. M. (2024). Charged anisotropic composite stellar object with strange, polytropic and gaseous matter. *New Astronomy*, *110*, 102229.

Olengeile, L., Sunzu, J. M., & Mkenyeleye, J. M. (2023). Three-layered super dense star with charged anisotropic fluid. *New Astronomy*, *100*, 102002.

Oppenheimer, J. R., & Volkoff, G. M. (1939). On massive neutron cores. *Physical Review*, *55*(4), 374.

Pant, K., & Fuloria, P. (2021). The comparative analysis of dense stellar models governed by quadratic and linear equations of state. *New Astronomy*, *84*, 101509.

Patel, R., Ratanpal, B. S., & Pandya, D. M. (2023). New charged anisotropic solution on paraboloidal spacetime. *Astrophysics and Space Science*, *368*(7), 58.

Rej, P., Bhar, P., & Govender, M. (2021). Charged compact star in f (R, T) gravity in Tolman–Kuchowicz spacetime. *The European Physical Journal C*, *81*(4), 316.

Rincón, Á., Panotopoulos, G., & Lopes, I. (2023). Anisotropic Quark Stars with an Interacting Quark Equation of State within the Complexity Factor Formalism. *Universe*, *9*(2), 72.

Ruderman, M. (1972). Pulsars: structure and dynamics. *Annual Review of Astronomy and Astrophysics*, *10*(1), 427-476.

Singh, K. N., Maurya, S. K., Dutta, A., Rahaman, F., & Aktar, S. (2021). Quark stars in 4-dimensional Einstein–Gauss–Bonnet gravity. *The European Physical Journal C*, *81*(10), 909.

Sokolov, A. I. (1980). Phase transitions in a superfluid neutron liquid. *Journal of*

*Experimental and Theoretical Physics,* 79(4), 1137.
Sunzu, J. M., Maharaj, S. D., & Ray, S. (2014). Quark star model with charged anisotropic matter *Astrophysics and Space Science*, *354*, 517-524.
Sunzu, J. M., & Mathias, A. V. (2022). A neutral stellar model with quadratic equation of state *Indian Journal of Physics*, *96*(14), 4059-4069.
Sunzu, J. M., & Thomas, M. (2018). New stellar models generated using a quadratic equation of state. *Pramana*, *91*, 1-10.
Tangphati, T., Panotopoulos, G., Banerjee, A., & Pradhan, A. (2023). Charged compact stars with color–flavor-locked strange quark matter in f (R, T) gravity. *Chinese Journal of Physics*, *82*, 62-74.
Tello-Ortiz, F., Maurya, S. K., & Gomez-Leyton, Y. (2020). Class I approach as MGD generator. *The European Physical Journal C*, *80*(4), 324.
Thirukkanesh, S., Sharma, R., & Das, S. (2020). Model of a static spherically symmetric anisotro-pic fluid distribution in paraboloidal spacetime admitting a polytropic equation of state. *The European Physical Journal Plus*, *135*(8), 1-14.
Usov, V. V. (2004). Electric fields at the quark surface of strange stars in the color-flavor locked phase. *Physical Review D*, 70, 067301.
Vaidya, P. C., & Tikekar, R. (1982). Exact relativistic model for a superdense star. *Journal of Astrophysics and Astronomy*, *3*, 325-334.